Cyclotron Emission and Thermalization of the CMB Spectrum

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If there was a weak magnetic field in the early universe, cyclotron emission could play an important role in the thermalization of the CMB. We study this process in the tightly coupled primordial electron-photon plasma and find that if the magnetic field is large enough so that the plasma effects allow emission of cyclotron photons, this process will wipe out deviations from the black body spectrum.

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The observed spectrum of the Cosmic Microwave Background radiation is beautifully fit by a black body spectrum. The FIRAS instrument on the COBE satellite measured this spectrum and found that deviations from a Planck spectrum are less than a few hundredth of a percent. In particular, the chemical potential potential is less than 9×10^{-5} [1]. This puts strong constraints on any non-thermal photon production mechanism, for example, the decay [2], [3], or annihilation [6] of relic particles after $z \sim 10^7$. After $z \sim 10^7$, the previously studied thermalization processes, i.e. bremsstrahlung and double Compton, can no longer significantly change the number of photons. As a lower limit, a value of $\sim 10^{-9}$ is expected for the chemical potential, in the absence of these processes. This a consequence of dissipation of the acoustic waves at small scales prior to the recombination [7].

There are indications that a primordial magnetic field is necessary to explain the observed magnetic fields in galaxies and galaxy clusters (see [8] and references therein for a review of the issues relevant to the primordial magnetic field). In the absence of any dynamo effect, the required field strength today is of the order of $10^{-9}G$ in the Mpc scales.

There are many proposed mechanisms for generating magnetic fields. They could be produced through QCD (e.g. [9], [10]) and Electroweak [11] phase transitions. The magnitude of the generated field depends on scale and the details of the phase transition model. Although the typical predictions are several orders of magnitude less than the required field strength ($\sim 10^{-20}G$, e.g. [12]) on the relevant scales, there are extreme models that can produce large enough field strengths [8]. However, note that the generated field can be significantly larger at small scales.

There are two main stages in the thermalization of the radiation field. The first stage is relaxation into a Bose-Einstein energy distribution which is mainly through Compton scattering of photons off electrons. The number of photons is fixed during this stage. This process, so-called Comptonization, is efficient for temperatures higher than 10 eV. The second stage is relaxation of the chemical potential which, in general, requires an increase in the number of photons. The most efficient processes in the early universe that can change the number of photons are bremsstrahlung and double Compton. In a universe with low barion density , double Compton is more important than bremsstrahlung but ceases to be efficient for $T < 1\,\mathrm{keV}$ [3].

We show that a weak magnetic field can change this picture for $T < 1\,\mathrm{keV}$. We see that if the cyclotron frequency is larger than the plasma frequency, cyclotron emission thermalizes the radiation spectrum. Finally we consider the observational and theoretical constraints on the possiblity of this effect and see that plausible levels of magnetic field strength may allow efficient cyclotron emission.

This phenomenon has been first considered in [4]. However, since the cyclotron absorption process was neglected, the phenomenon was interpreted as a process for generation rather than relaxtion of chemical potential. This problem was correctly pointed out in [5].

The rate of energy loss via cyclotron emission by non-relativistic electrons moving in magnetic field B can be obtained classically [13]

$$\frac{d\mathcal{E}}{dt} = \frac{2}{3}\frac{e^2\omega_c^2 v_\perp^2}{c^3}, \tag{1}$$

where $\omega_c = eB/(m_e c)$ is the cyclotron frequency and $v_\perp$ is the velocity of the electron, normal to the magnetic field direction.

In the non-relativistic limit, almost all the emitted photons have the frequency ω_c and thus, the rate of photon production per unit volume, ϕ, can be obtained using Eq. (1)

$$\phi = \frac{2}{3}\frac{n_e e^3 B < v_\perp^2 >}{\hbar m_e c^4} = \frac{4}{3}\frac{n_e e^3 B k_B T}{\hbar m_e^2 c^4}, \tag{2}$$

where n_e is the electron number density and T is the temperature of the electron gas.

In the rest of the paper, we are going to set $k_B = \hbar = c = 1$.

The presence of photons in the environment can enhance the photon production mechanism through stimulated emission. Also photons can get absorbed by the rotating electrons. These processes can be expressed via [14]

$$\dot{n}(E_c) = \sum_{\{E\}} \mathcal{A}(1 + n(E_c))n_e(E + E_c) - \mathcal{B}n(E_c)n_e(E). \tag{3}$$

Here, $n(E)$ is the photon occupation number, $E_c = \omega_c$ is the energy of the cyclotron photons and $\mathcal{A}$ & $\mathcal{B}$ are Einstein coefficients. The sum, is over the energy states of the of electrons (Landau levels, in this case).

We note that, since for a Planckian distribution

$$n_{Pl}(E) = \frac{1}{\exp(E/T) - 1}, \tag{4}$$

$\dot{n}$ must vanish, (if the electrons have the same temperature as photons, which is the case before matter-radiation decoupling , when Compton scattering is efficient) we must have

$$\sum_{\{E\}} \exp(E_c/T)\mathcal{A}n_e(E + E_c) = \sum_{\{E\}} \mathcal{B}n_e(E). \tag{5}$$

Since we are assuming that Comptonization is efficient, the only free parameters in the spectrum of photons are the temperature, T, and the chemical potential, μ. Alternatively, they can be replaced by the total number density, N, and the energy density u, of the photon gas. As a result, we can integrate Eq. (3) over the phase space to obtain the total photon injection rate, without losing any information. Multiplying this by E_c, gives the energy injection rate.

$$\dot{N} = 2\int d^3p \sum_{\{E\}} \mathcal{A}n_e(E + E_c)(1 + n(E_c) - \exp(E_c/T)n(E_c)). \tag{6}$$

The factor in front of the bracket is the photon production rate for zero photon occupation number, which is the same as ϕ in Eq.(2). Plugging in a Bose-Einstein energy distribution for $n(E_c)$, and assuming $\mu, E_c \ll T$, we end up with

$$\dot{N} = -\frac{\phi\mu}{E_c - \mu}, \tag{7}$$

and the energy production rate per volume

$$\dot{u} = E_c\dot{N} = -\frac{\phi E_c \mu}{E_c - \mu}. \tag{8}$$

For $\mu \ll T$, energy and number density of photons can be written as

$$u \simeq \frac{T^4}{\pi^2}[\frac{\pi^4}{15} + 7.212\frac{\mu}{T}],$$
$$N \simeq \frac{T^4}{\pi^2}[2.404 + \frac{2\pi^2}{3}\frac{\mu}{T}]. \tag{9}$$

To study the relaxation process, we replace T by $T+\theta$ and consider $\theta \ll T$ as the time dependent perturbation of the temperature, while T is constant. To the first order in θ and μ, we find

$$\delta u = \pi^{-2}T^3[4\alpha\theta + \beta\mu], \; \delta N = \pi^{-2}T^2[3\gamma\theta + \delta\,\mu] \tag{10}$$

where

$$\alpha = \pi^4/15, \; \beta \simeq 7.212, \; \gamma \simeq 2.404, \; \delta = 2\pi^2/3, \tag{11}$$

are the numerical coefficients in Eq. (9).

Now, we can use Eqs. (7)& (8) to get the time derivatives of δN and δu in Eq. (10). We are interested in the relaxation of μ, for which we find

$$\dot{\mu} = -[\frac{4\alpha\pi^2}{4\alpha\delta - 3\beta\gamma}]\frac{\phi\mu}{T^2(E_c - \mu)}. \tag{12}$$

Note that, to the first order, θ does not appear in Eq. (12). The reason is that, in the absence of cyclotron emission, μ and θ, both remain constant. Also, $\mu \leq 0$ to have a finite photon occupation number at all energies. Eq. (12) is simplified by introducing the following variables

$$\tilde{\mu} = -\frac{\mu}{E_c}, \; t_c = T^2E_c\phi^{-1}[\frac{4\alpha\pi^2}{4\alpha\delta - 3\beta\gamma}]^{-1}. \tag{13}$$

In terms of these variables, Eq. (12) takes the form

$$\dot{\tilde{\mu}} = -\frac{\tilde{\mu}}{t_c(1 + \mu)}, \tag{14}$$

with the solution

$$\tilde{\mu} + \ln\tilde{\mu} = -t/t_c + \text{const.} \tag{15}$$

We see that, for $\tilde{\mu} < 1$ ($|\mu| < E_c$), there is an exponential decay with the characteristic time t_c. In the radiation dominated era, in a cosmological scenario, the value t_c is *

$$t_c = 3.346(\frac{m_eT^3}{\alpha_e n_e m_{Pl}})H^{-1} = 1.723 \times 10^{-10}H^{-1}. \tag{16}$$

where, $m_{Pl} = G^{-1/2}$ and $\alpha_e = e^2$ are the Planck's mass and the fine structure constant. It is amazing that the ratio of t_c to the Hubble time is so small and also independent of time and the magnetic field strength. However,

*Assuming $T_{\rm CMB} = 2.73$K, $h_{100} = 0.7, g_{\rm eff} = 3.36$ and $\Omega_b = 0.04$.

this is not un-physical, since as B goes to zero, the cyclotron energy, E_c, becomes smaller than μ and we are not in this regime any more. Since stimulated emission is proportional to the occupation number which goes as E_c^{-1} at small energies, it cancels the E_c that appears in ϕ in Eq. (2), thus the equilibrium time is independent of the magnetic field strength.

More interesting is the case of $\tilde{\mu} > 1$ ($|\mu| > E_c$). In this case, $\dot{\tilde{\mu}} \simeq t_c^{-1}$ is almost constant and so the relaxation time is proportional to the initial value of $\tilde{\mu}$.

$$t_{rel} = (\frac{-\mu}{E_c})t_c = 2.22(Ht_c)(\frac{-\mu}{T})\beta_M^{-1/2}(\frac{T}{m_e})^{-1}H^{-1}$$
$$= (6.204 \times 10^{-2} H^{-1})(\frac{-\mu}{T})(\frac{\beta_M}{10^{-5}})^{-1/2}T^{-1}(eV), \quad (17)$$

where

$$\beta_M \equiv \frac{B^2}{8\pi\rho}, \quad (18)$$

and ρ is the energy density of the universe and so β_M is the fraction of the total energy density in the magnetic field (Note that β_M remains constant in the radiation dominated era). At the end of this time, the chemical potential decays in a time scale, much shorter than the Hubble time.

Since the decay is linear, rather than exponential, this relaxation process behaves differently from other processes (specifically bremsstrahlung and double Compton): either μ/T is basically constant (if $Ht_{rel} > 1$) or it is completely suppressed ($Ht_{rel} < 1$). Eq. (18) implies that even a dynamically negligible primordial magnetic field ($B^2 \sim 10^{-6}\rho$ which is $\sim 10^{-9}G$ today, if the flux remains frozen) can completely suppress even a significant deviation from the Planck spectrum in the radiation dominated era.

The natural frequency of the primordial plasma sets a lower bound for the frequency of propagating photons. As a result, no photon can be emitted via cyclotron emission if ω_c falls below the plasma frequency, $\omega_p = \sqrt{4\pi n_e e^2/m_e}$. Therefore this thermalization process works only if

$$\frac{\omega_p^2}{\omega_c^2} = \frac{4\pi n_e m_e}{B^2} = 0.452\, \beta_M^{-1}(\frac{n_e}{T^3})(\frac{m_e}{T})$$
$$= 2.575 \times 10^{-5}\, \beta_M^{-1}\, T^{-1}(\text{eV}) \, < \, 1, \quad (19)$$

which yields

$$\beta_M = \frac{B^2}{8\pi\rho} > 2.575 \times 10^{-5} T^{-1}(\text{eV}). \quad (20)$$

or in terms of the magnetic field strength at the present time

$$B > 2.3 \times 10^{-8} T^{-1/2}(\text{eV})\text{G} \quad (21)$$

It is reasonable to assume that the maximum value of $-\mu/T$, as a result of a non-thermal process, is close to one, since μ and T are affected in similar ways by non-thermal processes (see Eq.(10)). With this assumption, combining Eqs. (17) & (20) yields

$$Ht_{rel} < 3.866 \times 10^{-2} T^{-1/2}(\text{eV}) < 1, \quad (22)$$

for $T > 10^{-3}$eV, which is clearly the case in the radiation dominated era. This leads to the main conclusion of this letter: *if the condition of Eq. (20) is satisfied, any deviation from the Planck spectrum will be completely suppressed.* This will be basically indpendent of the origin or the magnitude of this deviation.

The strongest observational constraint on the magnitude of a primordial magnetic field comes from the spectrum of the CMB fluctuations [15]. The current upper limit on β_M is about 10^{-10} at the comoving scale of $\sim 1\,$Mpc. However, the actual strength of the magnetic field can be significantly larger at smaller scales. For example, with the random dipole approximation [8], $\beta_M \propto r^{-3}$ and so we see that the observational constraint hardly gives us any information about the possibility of cyclotron thermalization process if the field extends down to the scale of $r \sim 10\,$kpc. If the actual magnitude of the field is anywhere close to the observational limit at this scale, it is likely that β_M will satisfy Eq.(20) at smaller scales and the cyclotron emission will suppress any chemical potential in the CMB.

A direct method has been suggested in [5] to set an upper limit on the field strength at small scales. The method is based on the fact that the dissipation of small scale magnetic field can alter the chemical potential of the CMB. The current upper limit on the chemical potential [1], yields an upper limit on the magnetic field, $B < 3 \times 10^{-8}$G at the comoving scale of ~ 400pc. It is amazing that this limit is so close to the lower limit for B in Eq. (21), for the cyclotron emission to be efficient. The implication is that if Eq. (21) is satisfied, cyclotron emission suppresses any chemical potential, while if Eq. (21) is not satisfied, the induced chemical potential is less than the observational limit. Therefore, both cases are consistent with the observations.

The theoretical estimate for the magnitude of β_M, generated in the QCD phase transition at $T_c \sim 150$MeV, which relies on the presence of hydrodynamic instabilities produced by expanding bubble walls, is about 10^{-29} at a 10Mpc comoving scale [12]. This translates to $\beta_M \sim 10^{-8}$ at 1pc, the comoving Hubble radius at the phase transition. This value is barely enough to satisfy Eq. (20), at $T = 1$keV, where double-Compton ceases to be efficient but falls below the the required limit for lower temperatures.

On the other hand, another way of generating magnetic field is via Electroweak phase transition [11]. The maximum value of β_M generated in this case can be as large as 10^{-3} which does satisfy Eq.(20).

In summary, we introduce cyclotron emission as a potential thermalization process of the CMB spectrum. We found that the process of relaxation of the chemical potential, in a regime that Comptonization is efficient, is linear instead of exponential for large deviations of the chemical potential from zero. As a result, the relaxation time is proportional to the original value of the chemical potential. Requiring that the plasma effects do not suppress this process sets a lower limit on the magnitude of the magnetic field. While observational constraints are not conclusive about if the primordial magnetic field is larger than this lower limit or not, there are theoretical estimates that suggest magnetic fields large enough for cyclotron thermalization process to be efficient, are produced in the early universe.

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